Measuring the characteristic function of the work distribution

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We propose an interferometric setting for the ancilla-assisted measurement of the characteristic function of the work distribution following a time-dependent process experienced by a quantum system. We identify how the configuration of the effective interferometer is linked to the symmetries enjoyed by the Hamiltonian ruling the process and provide the explicit form of the operations to implement in order to accomplish our task. We finally discuss two physical settings, based on hybrid opto-/electro-mechanical devices, where the theoretical proposals discussed in our work could find an experimental demonstration.

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Thermodynamics is one of the pillars of physical, chemical and biological sciences. Its principles are able to predict and explain the occurrence and efficiency of complex chemical reactions and biological processes. In physics, the conduction of heat across a medium or the very concept of the arrow of time are formulated thermodynamically. In information theory, the definitions of information and entropy are also given in thermodynamical terms and explicitly related to this framework. Even more, the tightness of the link between information and thermodynamics can be deduced from the thermodynamical interpretation of the landmark embodied by Landauer's erasure principle [1].

The outstanding levels of dexterity and reliability that characterise the current experimental ability to control and guide processes at the microscopic scale opens up a series of tantalising questions, the most pressing one being probably the following: *what happens to the principles of thermodynamics when we deal with the quick, non-quasistatic dynamics of small quantum systems brought out of equilibrium?* An invaluable tool for the formulation of an answer in this sense has been provided with the formulation of non-equilibrium fluctuation relations and their subsequent extension to the quantum formalism [2, 3], which has very recently enabled interesting investigations at the border of quantum physics, thermodynamics, and information theory [4], including the experimental proposal of building blocks for quantum thermal machines [5], the study on the connection between quantum fluctuation relations and critical phenomena in many-body quantum systems [6, 7], the verification of key features of the quantum thermodynamical framework as embodied by the Jarzynski equality [8–10], and the extension to open-system dynamics [11].

It is precisely the Jarzynski identity to serve the motivations behind our work: the key point for its verification and use, both in the classical and quantum formulations [10, 11], is the determination of the work distribution following a process undergone by a system. It is thus central to the potential application and development of the investigation on non-equilibrium quantum dynamics. To the best of our knowledge, the most advanced experimental scheme for the determination of work distribution following a quantum process relies on hard projections onto energy eigenstates of the system directly, which can be of significant practical difficulty [8]. It is thus important to devise strategies for reaching this outstanding goal.

Our work moves along these lines to propose a way to infer the quantum statistics of a work distribution by relying on a well-known interferometric approach that delegates the retrieval of the information we are after to routine measurements performed on a finite-size ancilla. In fact, we demonstrate how a qubit-assisted Ramsey-like scheme is effective in fully determine the characteristic function of the work distribution following a general quantum process. This is enough information to access the work distribution itself and, thus, employ the framework of fluctuation relations to an out-of-equilibrium configuration. We clearly identify the relation between symmetries that the quantum process might enjoy and the corresponding Ramsey interferometer. Although our scheme does not rely on the details of a specific experimental setting and can thus be applied to a vast range of interesting physical situations (cf. Ref. [12] for a related analysis applied to an ion-trap setting), as an illustration we apply it to the case of a (micro-/nano-)mechanical oscillator coupled to an effective two-level system and undergoing a process that displaces its state in the phase space, thus showing the effectiveness of our scheme to a situation of current strong experimental interest. We believe that designing experimentally viable ways to access quantum statistics of non-equilibrium dynamical processes can embody a significant step towards the experimental grounding and the development of this fascinating area of investigation and the spurring of potential ramifications in areas such as quantum control and foundations of quantum mechanics [4, 13, 14].

Quantum fluctuation relations: a brief review.– In order to provide a self-contained discussion, here we give a brief summary of the quantum fluctuation formalism that will be used in the rest of the manuscript. We consider a process described by a Hamiltonian $\hat{\mathcal{H}}(\lambda(t))$ that depends on a so-called *work parameter* $\lambda(t)$, which is assumed to be externally controlled. A system S is assumed in contact with a reservoir and initialised in a thermal equilibrium state $\rho_S^{th}(\lambda_0) = e^{-\beta\hat{\mathcal{H}}(\lambda_0)}/\mathcal{Z}(\lambda_0)$ at inverse temperature β and work parameter $\lambda(0^-) = \lambda_0$ [here $\mathcal{Z}(\lambda) = \mathrm{Tr}e^{-\beta\hat{\mathcal{H}}(\lambda)}$ is the partition function]. At $t = 0^+$, we

detach S from the reservoir and perform the *process*, which consists in changing the work parameter to its final value $\lambda(\tau) = \lambda_\tau$. It is now convenient to decompose the two Hamiltonians connected by the process as $\hat{\mathcal{H}}(\lambda_0) = \sum_n E_n(\lambda_0) |n\rangle \langle n|$ and $\hat{\mathcal{H}}(\lambda_\tau) = \sum_m E'_m(\lambda_\tau) |m\rangle \langle m|$, where $(E_n, |n\rangle)$ $[(E'_m, |m\rangle)]$ is the $n^{\rm th}$ $[m^{\rm th}]$ eigenvalue-eigenstate pair of the initial [final] Hamiltonian. The corresponding work distribution can be written as [3]

$$P_{\rightarrow}(W) := \sum_{n,m} p_n^0\, p_{m|n}^{\tau} \delta\left[W - (E'_m - E_n)\right]. \tag{1}$$

Here, we have introduced the probability p_n^0 that the system is found in state $|n\rangle$ at time $t = 0$ and the conditional probability $p_{m|n}^{\tau}$ to find it in state $|m\rangle$ at time $t = \tau$ if it was in $|n\rangle$ at $t = 0$ and has evolved under the action of the propagator $\hat{U}(\tau, 0)$. Eq. (1) encompasses both the thermal statistics of the initial thermal state (given by p_n^0) and the fluctuations arising from quantum measurement statistics (given by $p_{m|n}^{\tau}$). The role of $P_{\rightarrow}(W)$ becomes immediately evident when considering the Tasaki-Crooks relation $P_{\rightarrow}(W)/P_{\leftarrow}(-W) = e^{\beta(W-\Delta F)}$ [3, 15], which states that the ratio between the forward work distribution $P_{\rightarrow}(W)$ and the backward one $P_{\leftarrow}(-W)$, obtained implementing the process $\lambda_\tau \rightarrow \lambda_0$, starting from $\rho_S^{th}(\lambda_\tau)$ and evolving through $U^{\dagger}(\tau, 0)$, is related to the difference in the equilibrium free-energy ΔF of the system. For our purposes, it is convenient to define the characteristic function of the work distribution [16]

$$\begin{aligned}\chi(u, \tau) &= \int \mathrm{d}W\, e^{iuW} P_{\rightarrow}(W) \\ &= \mathrm{Tr}\left[\hat{U}^{\dagger}(\tau, 0) e^{iu\hat{\mathcal{H}}(\lambda_\tau)} \hat{U}(\tau, 0) e^{-iu\hat{\mathcal{H}}(\lambda_0)} \rho_{\rm G}(\lambda_0)\right].\end{aligned} \tag{2}$$

From Eq. (2), the Jarzynski equality [10] is found by introducing the parameter $u = i\beta$ in such a way that

$$\chi(i\beta, \tau) = \langle e^{-\beta W}\rangle = e^{-\beta\Delta F}. \tag{3}$$

Eq. (3) demonstrates the central role played by W (and, in turn, $P_F(W)$ and its characteristic function) in the determination of the equilibrium properties of a system driven by a process. In what follows, we shall illustrate a protocol for the interferometric determination of $\chi(u, \tau)$, which would then enable the convenient evaluation of the figures of merit discussed above.

A simple illustrative case.– In order to fix the ideas before attacking the general protocol and illustrate the idea behind our proposal in an easy-to-grasp situation, we consider a Hamiltonian for S having the form

$$\hat{\mathcal{H}}_S(t) = g(\lambda_t)\hat{h}, \tag{4}$$

with $\hat{h}$ being the operatorial part of the Hamiltonian that remains unchanged irrespective on the process implemented for the change of the work parameter (notice that, for easiness of notation, from now on we will replace $\lambda(t) \rightarrow \lambda_t$) and specified fully by the assignment of the function $g(\lambda_t)$. As a consequence, the system Hamiltonian commutes with itself and the time-evolution operator $\hat{U}(\tau, 0) = e^{-i\int_0^{\tau}\hat{\mathcal{H}}_S(t)dt} = e^{-i\hat{h}\int_0^{\tau} g(\lambda_t)dt}$ at all instant of time. That is

$$[\hat{\mathcal{H}}_i, \hat{\mathcal{H}}_f] = 0 \quad \text{and} \quad [\hat{U}(\tau, 0), \hat{\mathcal{H}}_i] = [\hat{U}(\tau, 0), \hat{\mathcal{H}}_f] = 0 \tag{5}$$

with $\hat{\mathcal{H}}_i \equiv \hat{\mathcal{H}}_S(0) = g(\lambda_0)\hat{h}$ $[\hat{\mathcal{H}}_f \equiv \hat{\mathcal{H}}_S(\tau) = g(\lambda_\tau)\hat{h}]$. Due to such symmetries, it is straightforward to check that the characteristic function of the work distribution simplifies to

$$\begin{aligned}\chi_{\rm simp}(u) &= \mathrm{Tr}\left[\hat{U}^{\dagger}(\tau, 0) e^{iu\hat{\mathcal{H}}_f(\lambda_\tau)} \hat{U}(\tau, 0) e^{-iu\hat{\mathcal{H}}_i(\lambda_0)} \rho(\lambda_0)\right] \\ &= \mathrm{Tr}\left[e^{i(\hat{\mathcal{H}}_f - \hat{\mathcal{H}}_i)u} \rho(\lambda_0)\right].\end{aligned} \tag{6}$$

The interpretation of this result is straightforward: the characteristic function is fully determined by the changes induced by the process responsible for the variation of the work parameter in the Hamiltonian of the system S. In turn, this allows us to make a significant step forward in the illustration of the proposed interferometric setting. Indeed, let us now introduce a simple qubit system playing the role of an ancilla A whose role is to assist in the measurement of $\chi_{\rm sump}$. Moreover, we consider the system-ancilla evolution $\hat{G}(u)\hat{V}(u)$, where $\hat{V}(u) = e^{-i\hat{\mathcal{H}}_i u} \otimes \hat{\mathbb{1}}_A$ is a local transformation on the S Hilbert space and $\hat{G}(u)$ is the controlled A-S gate

$$\hat{G}(u) = \hat{\mathbb{1}}_S \otimes |0\rangle_A\langle 0| + e^{-i(\hat{\mathcal{H}}_f - \hat{\mathcal{H}}_i)u} \otimes |1\rangle_A\langle 1|, \tag{7}$$

which applies the transformation $e^{-i(\hat{\mathcal{H}}_f - \hat{\mathcal{H}}_i)u}$ to the state of S only when the ancilla is in $|1\rangle_A$ and leaves it unaffected otherwise. Inspired by Ramsey-like schemes for parameter estimation [17, 18], our protocol proceeds as follows: We prepare the ancilla in $|0\rangle_A$ and apply a Hadamard transform $\hat{H} = (\hat{\sigma}_x + \hat{\sigma}_z)/\sqrt{2}$ [24] that changes it into the eigenstate of the x-Pauli matrix $|+\rangle_A = (|0\rangle_A + |1\rangle_A)/\sqrt{2}$. We then apply the sequence $\hat{G}(u)\hat{V}(u)$ on the system-ancilla state $|+\rangle_A\langle +| \otimes \rho_S^{th}$ and subject the ancilla to a second Hadamard transform [cf. Fig. 1]. The controlled system-ancilla gate $\hat{G}(u)$ in general establishes quantum correlations between the qubit A and S. The fingerprint of this is provided by the fact that information on the system can be retrieved from the ancilla. In fact, by tracing out the state of S, we find

$$\begin{aligned}\rho_A &= \mathrm{Tr}_S[\hat{H}\hat{G}(u)\hat{V}(u)(|+\rangle_A\langle +| \otimes \rho_S^{th})\hat{V}^{\dagger}(u)\hat{G}^{\dagger}(u)\hat{H}] \\ &= \frac{1}{2}(\hat{\mathbb{1}}_A + \alpha\hat{\sigma}_z - i\nu\hat{\sigma}_y)\end{aligned} \tag{8}$$

with $\alpha = \mathrm{Re}\chi_{\rm simp}$ and $\nu = \mathrm{Im}\chi_{\rm simp}$. This proves our claim and the effectiveness of our protocol for the reconstruction of the characteristic function of the work distribution: the tomography of populations and coherences of the ancilla state enables the full determination of $\chi_{\rm simp}(u)$.

General protocol.– We now relax the assumption made previously on the form of the Hamiltonian of the system and consider the general case embodied by the conditions

$$[\hat{\mathcal{H}}_i, \hat{\mathcal{H}}_f] \neq 0 \qquad \text{and} \qquad [\hat{U}(\tau, 0), \hat{\mathcal{H}}_{i(f)}] \neq 0. \tag{9}$$

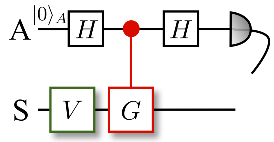

FIG. 1: (Color online) Quantum circuit illustrating the simplified protocol for the measurement of the characteristic function of the work distribution through a single-probe interferometer. The ancilla A is a qubit initialised in $|0\rangle_A$ and undergoes a Hadamard gate $\hat{H}$. The system S is prepared in a thermal state ρ_S^{th} and is subjected to the local transformation $\hat{V}$. The controlled gate $\hat{G}$, followed by a second Hadamard gate on A, implement our protocol. We refer to the body of the manuscript for the explicit form of the gates (whose dependence on u has been omitted for easiness of notation).

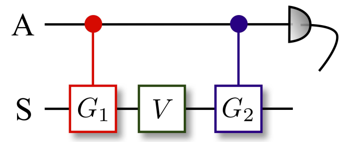

FIG. 2: (Color online) Quantum circuit illustrating the protocol for the measurement of the characteristic function of the work distribution through a single-probe interferometer in the most general case. In order to ease the visualization of the scheme, here we have absorbed the Hadamard gate $\hat{H}$ that should be used as the start and end step of the protocol into the preparation of the ancilla state and the basis onto which the measurements should be performed. The system S is assumed to be prepared in a thermal state ρ_S^{th}. The sequence $\hat{G}_2\hat{V}\hat{G}_1$ implements our proposal.

Correspondingly, the characteristic function takes the full form given in Eq. (2). In this case, the general interferometric approach illustrated above still applies, the only difference being the form of the controlled operation to be applied on the state of the system. More explicitly, we now need to implement the joint A-S gate

$$\hat{G}(u,\tau) = \hat{U}(\tau,0)e^{-i\hat{\mathcal{H}}_i u}\,|0\rangle_A\langle 0| + e^{-i\hat{\mathcal{H}}_f u}\hat{U}(\tau,0)\,|1\rangle_A\langle 1|\,, \quad (10)$$

which can be broken down into the concatenation of a local transformation on S and two A-controlled gates as $\hat{G}(u,\tau) = \hat{G}_2(u,\tau)\hat{V}(u)\hat{G}_1(u,\tau)$ with

$$\begin{aligned}\hat{G}_1(u,\tau) &= \hat{\mathbb{1}}_S \otimes |0\rangle_A\langle 0| + e^{i\hat{\mathcal{H}}_i u}\hat{U}(\tau,0)\otimes|1\rangle_A\langle 1|\,,\\ \hat{V}(u) &= \hat{\mathbb{1}}_A \otimes e^{-i\hat{\mathcal{H}}_i u}e^{i\hat{\mathcal{H}}_f u}, \qquad (11)\\ \hat{G}_2(u,\tau) &= \hat{U}(\tau,0)e^{i\hat{\mathcal{H}}_f u}\otimes|0\rangle_A\langle 0| + \hat{\mathbb{1}}_S\otimes|1\rangle_A\langle 1|\,.\end{aligned}$$

The sequence of operations to apply to the S-A system is illustrated in Fig. 2, which provides the quantum logic circuit for the interferometric protocol addressing the most general process undergone by the system. Using the same preparation of the ancilla as above and the two Hadamard transforms, we obtain a reduced state of A identical to Eq. (8) with the replacements $\alpha \to \mathrm{Re}\chi(u,\tau)$ and $\nu \to \mathrm{Im}\chi(u,\tau)$ with $\chi(u,\tau)$ as in Eq. (2). An appropriate sequence of measurements performed on the ancilla state only is thus able to provide information on the work distribution following the process undergone by S.

Potential physical examples.–Among others [12], two explicit situations of enormous current experimental interest can well be used in order to illustrate the main findings of our work. They are both based on hybrid coupling configurations between an effective two-level system and a mechanical oscillator, which can be either microscopic (in a hybrid cavity optomechanics setup) or nanoscopic (for an electromechanical implementation). In what follows, we briefly show how to achieve the effective Hamiltonians regulating the processes that we have in mind in both such scenarios and illustrate the principles of our proposal by explicitly calculating the characteristic function following it.

We start from a microscopic setting, where we consider a three-level atom in a Λ configuration coupled to a single-mode optical cavity that is pumped by a laser field at frequency ω_p and having a movable mirror. The atom is driven by a second field at frequency ω_i that enters the cavity radially [cf. Fig. 3 **(a)**]. The logical states $\{|0\rangle, |1\rangle\}$ of the ancilla system A are encoded in the fundamental atomic doublet ($|e\rangle$ being the common excited state). The scheme includes the driving, at rate Ω, of the transition $|1\rangle\leftrightarrow|e\rangle$ by the external field at frequency ω_i. The transition $|0\rangle\leftrightarrow|e\rangle$ is guided by the cavity field (frequency ω_c), at rate g. Both the fields are detuned by δ from the excited atomic, while the detuning between cavity field and external pump is Δ=ω_c−ω_p.

System S is embodied by the movable mirror, which oscillates harmonically at frequency ω_S and is driven by the cavity field through radiation-pressure (at a rate χ) [19]. We assume large single-photon Raman detuning and negligible decay rate γ_e from the atomic excited state, so that $\delta \gg (\Omega, g) \gg \gamma_e$ and an off-resonant two-photon Raman transition is realized. We assume $\Delta \gg g, \chi$, so that both the atomic excited state and the cavity field are only virtually populated and they can be eliminated from the dynamics of the system, and move to an interaction picture defined by the operator $\omega_p\hat{c}^\dagger\hat{c} + \omega_i\,|e\rangle\langle e| + \omega_{10}\,|0\rangle_A\langle 0|$ with $(\hat{c},\hat{c}^\dagger)$ the bosonic operators of the cavity field.

We thus obtain the effective Hamiltonian [20] (we take units such that $\hbar$=1 throughout the paper) $\hat{\mathcal{H}}_{\mathrm{micro}} = \omega_S\hat{b}^\dagger\hat{b} + \lambda\,|1\rangle_A\langle 1|\otimes(\hat{b}^\dagger+\hat{b})$ with $\lambda = \chi g^2\Omega^2/\delta^2\Delta^2$ and $(\hat{b},\hat{b}^\dagger)$ the bosonic operators of the mechanical oscillator. Through the two-photon Raman transition, the virtual quanta resulting from the atom-cavity field interaction are transferred (by the bus embodied by the cavity field) to the mechanical oscillator. The state of the latter is correspondingly displaced in its phase space, in a way that is controlled by the state of A. By driving the cavity with a bichromatic pump with frequencies $\omega_p(t)$ and $\omega_p(t) + \omega_m$ and relative phase $\phi(t)$, the effective coupling between the ancilla and the system becomes such that displacements in any direction of the phase space of the movable mirror can be arranged [21–23]. Moreover, considering a time-dependent amplitude of the driving field, we get $\lambda \to \lambda_t = \chi g^2\Omega^2(t)/\delta^2\Delta^2$, so that the model

$$\hat{\mathcal{H}}'_{\mathrm{micro}} = \omega_S\hat{b}^\dagger\hat{b} + \lambda_t\,|1\rangle_A\langle 1|\otimes(\hat{b}^\dagger e^{i\phi(t)} + \hat{b}e^{-i\phi(t)}) \qquad (12)$$

is finally obtained. As our examples aim to only have an illustrative character, we neglect the sources of damping and decoherence in the system, whose inclusion will require the extension of Eq. (2) to open-system dynamics. While this is possible due to the advances reported in [11], it goes significantly beyond the goals of this paper and will be left for a future investigation. However, it is important to stress that current progresses in the fabrication of mechanical oscillators allow for very small decoherence rates, while optical cavities with large quality factors are currently employed in optomechanical experiments [19], thus making a quasi-unitary picture plausible indeed. The state of the atomic system can be finally manipulated and reconstructed through an optical probe and employing standard tools in quantum optics.

A similar effective model is obtained by considering the second physical setting that we have in mind and shown in Fig. 3 **(b)**. This involves a nanomechanical oscillator (a *nano beam*) coupled capacitively to a Cooper-pair box (CPB) operating at the so-called charge degeneracy point [26], where the dynamics of the CPB can be approximated to that of a two-level system encoded in the space spanned by states $|a_\pm\rangle$ that are symmetric and antisymmetric superpositions of states with exactly 0 and 1 excess Cooper pairs in the large superconducting island shown in Fig. 3 **(b)** and encode our ancilla. The natural Hamiltonian of the system reads $\hat{\mathcal{H}}_1 = \frac{(\hat{Q}-Q_g(t))^2}{2C_t} - E_J \cos\hat{\varphi} + \omega_S \hat{b}^\dagger \hat{b}$ with $\hat{Q}$ and $\hat{\varphi}$ the canonical charge and phase operator of the CPB, C_t the total capacitance of the island, $Q_g(t) = C_g V_g(t) + C_x V_x(t)$ the total gate charge, E_J the Josephson energy and Ω the frequency of the nanomechanical oscillator [26]. By defining $\hat{\Sigma}_x = |a_+\rangle\langle a_-| + |a_-\rangle\langle a_+|$, expanding $\hat{\mathcal{H}}_1$ in series of the ratio between the actual position of the mechanical oscillator and its equilibrium distance from the CPB (the amplitude of the oscillations is assumed to be small enough that only first-order terms are retained in such expansion) and adjusting the gate and driving voltages such that $Q_g(t) \simeq 0$, the interaction Hamiltonian of the system can be cast into the form

$$\hat{\mathcal{H}}_{\text{nano}} = \omega_S \hat{b}^\dagger \hat{b} + \lambda_t (\hat{b} + \hat{b}^\dagger) \otimes \hat{\Sigma}_x \tag{13}$$

(the explicit form of λ_t is inessential for our tasks) [25]. The state of the ancilla can be processed by tuning the gate voltage $V_g(t)$ and the Josephson energy, while single-qubit measurements in this scenario are performed using single-electron transistors [26].

Both the models derived briefly above entail a conditional displacement of the harmonic oscillator. Actually, the phase-space displacement is precisely the time-dependent process that we choose for S. We thus take the coupling strength λ_t from $\lambda_0 = 0$ to λ_τ at the end of the process. In what follows, we concentrate on the model embodied by Eq. (12), which can be easily diagonalised by the means of appropriate, time-dependent Weyl displacement operators. A lengthy yet straightforward calculation based on standard phase-space methods allows us to fully evaluate the density matrix of the ancilla associated with the process experienced by the harmonic oscillator and following the protocol that we have suggested, which can be conveniently cast into the form

$$\begin{aligned}\chi(u,\tau) &= e^{-2iu\lambda_\tau \mathrm{Im}[\gamma_\tau e^{i\omega_S \tau}]} \mathrm{Tr}_S\,[e^{iu(\omega_S \hat{b}^\dagger \hat{b} + \lambda_\tau (\hat{b}^\dagger + \hat{b}))} \rho_S^{th}] \\ &= e^{i\frac{\lambda_\tau^2}{\omega_S^2}\sin(\omega_S u) - 2iu\lambda_\tau \mathrm{Im}[\gamma_\tau e^{i\omega_S \tau}] - |\mu_\tau(u)|^2/2} \sum_n \frac{(\overline{n} e^{i\omega_S u})^n}{(\overline{n}+1)^{n+1}} \mathcal{L}_n^{(0)}(|\mu_\tau(u)|^2),\end{aligned} \tag{14}$$

where $\mathcal{L}_n^{(p)}(x)$ are associated Laguerre polynomials of order (n,p) and argument x and we have introduced the parameters $\mu_\tau(u) = \lambda_\tau(1 + e^{i\omega_S u})/\omega_S$, $\gamma_\tau = \int^\tau \lambda_t e^{-i\omega_S t} dt$, $\overline{n} = 1/(e^{\beta\omega_S} - 1)$. The details of this derivation, and an explicitly analysis of the characteristic function will be given in Ref. [27]. It is worth noticing that the protocol for the measurement of the characteristic function performed using the model in Eq. (13) is entirely equivalent to the above, modulo local unitary operations performed on the ancilla qubit [27].

Conclusions.–We have proposed an ancilla-assisted interferometric protocol for the measurement of the characteristic function of the work distribution corresponding to a process that is enforced on a system. The scheme requires both local operations performed on the oscillator and joint gates controlled by the state of the ancilla, and shares similarities with Ramsey-based strategies for parameter estimation. We have

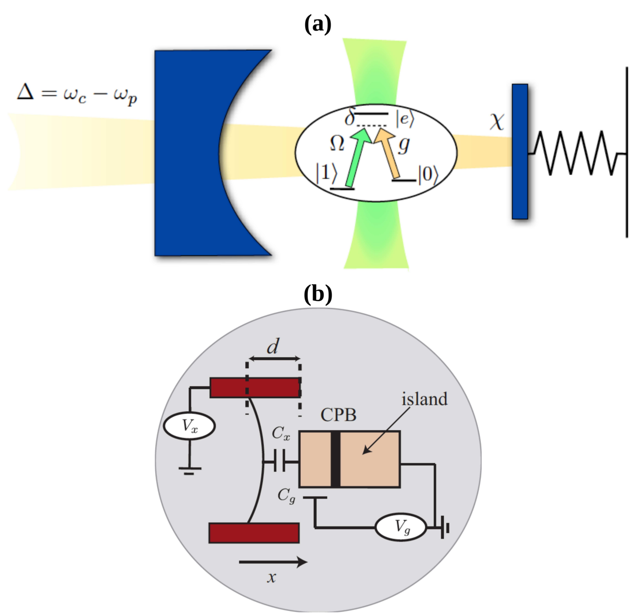

FIG. 3: (Color online) **(a)** Hybrid micro-optomechanical setting for the achievement of an effective Hamiltonian suitable for the verification of the protocol for the measurement of the characteristic function of the work distribution. The process is undergone by a system S embodied by the movable mirror of the optomechanical cavity. The ancilla A of our protocol is encoded in the ground-state doublet of a three-level atom. **(b)** Nano-mechanical version of the setup. We consider S as embodied by an electrically driven nanobeam (bias voltage V_x). The ancilla is a CPB coupled to S via the capacitance C_x. The state of the CPB is controlled by the gate voltage V_g (coupled to the box through the capacitance C_g) and the Josephson energy E_J. We work at the charge degeneracy point.

illustrated the working principles of our protocol by tackling the case of a mechanical oscillator undergoing a time-dependent displacement in phase space and coupled to an atomic ancilla, which embodies an interesting case for non-equilibrium quantum dynamics of current strong experimental interest. It should be noted, though, that our proposal bears no dependence on the specific experimental setting being considered and is widely applicable to any physical system allowing for a controllable and engineerable system-ancilla interaction and easily performable measurements on the latter (cf. [12] for an example based on an ion-trap setup).

As the characteristic function of the work distribution is a key element in the framework of quantum fluctuation relations for out-of-equilibrium system, designing experimentally viable strategies for its inference is an important step forwards in the grounding and development of out-of-equilibrium quantum thermodynamics. We believe that our proposal contributes to such a quest in a relevant way and opens up the possibility for an experimentally implementable verification of the connections between out-of-equilibrium quantum statistics and criticality in a quantum many-body system [6, 13, 18]. Very interesting routes for the application of our protocol include the study of quantum statistical properties of quantum thermal machines [14].

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